

2.3.3 Bruxelles

Marie-Laurence De Keersmaecker¹⁰ et Isabelle Thomas¹¹

I. Introduction

The present contribution aims at analyzing the structure of the intra-urban built-up areas (1) of Brussels taking the built-up structure into account, (2) using different fractal measurement methods and (3) linking the obtained values to several variables commonly used in spatial urban economics, urban geography and land use planning. Let us remind that Brussels is an agglomeration, where urbanistic rules are rather free, making it possible for each urban residential ward to have its own urban dimension(s). Specifically, this research focuses on whether different intra-urban patterns (historical center, planned urbanization, social housing, etc.) can have different and/or unique fractal dimensions. It also attempts to see how far fractal dimension is statistically related to variables such as rent, household income, distance to CBD, etc. In other words, is fractal dimension a useful index for distinguishing urban wards? This paper attempts to draw two types of conclusions. On the one hand, it aims to link the empirical fractal results to geographical and economical theories but also to urban planning and land use (geographical conclusions). On the other hand, it aims to test whether fractal indices enable one to improve the analysis of the structure of the urban wards, their morphology, barriers effects, etc. (urbanistic results).

The paper is organized as follows. Section II briefly explains the data processing step. Section III discusses the computational results in terms of exploratory data analysis and Section IV in terms of bivariate relationships. Results are interpreted in an urban territorial development context. Conclusions, perspectives and remarks are included in Section V.

II. Methodology

Our intention is here to use the fractal dimensions (D) obtained on surfaces and on borders by means of two different methods described earlier in this report (*diDil* and *Cor*) in the case of the built-up areas in Brussels.

Brussels is the capital city of Belgium, located almost in the center of the country. Defining its limits is an objective on its own (see e.g. GEMACA 1995, Thomas *et al.* 2000, Vanderhaegen *et al.*, 1996): the city sprawls far beyond its original boundaries. In the administrative sense, the Brussels Capital-Region (B.C.R.) is one of the three Regions of the Belgian federal state. Spatially, it corresponds to the enlarged city center (19 communes) and hence excludes recent peripheral wards that mainly extend to the two other administrative and linguistic regions. This paper only refers to the city defined by the limits of the B.C.R. It corresponds to 954,000 inhabitants and 16,138 hectares (in 2001). Ignoring the peripheral communes means better isolating urban from periurban land uses.

The necessary input of our analysis is a **city map** representing built-up areas. The C.I.R.B. (Centre Informatique pour la Région Bruxelloise) has developed a comprehensive G.I.S. for

¹⁰ Unité d'Urbanisme et de Développement territorial de l'U.C.L., Louvain-la-Neuve, Belgique

¹¹ Département de Géographie de l'U.C.L., Louvain-la-Neuve, Belgique, C.O.R.E., Louvain-la-Neuve, Belgique, Fonds National de la Recherche Scientifique, Bruxelles

Brussels Capital-Region. The layer corresponding to the built-up areas is the only one taken into consideration in this analysis (Figure 3). Each pixel in the map represents an area of 2.5x2.5 meters on the ground. We are aware that each built-up information is binary (built-up, not built-up); no information is provided about the function of the building (hospital, plant or residence), nor about the type of green area (garden, public square, etc.). Moreover, there is no information about roads, parking spaces, rail-tracks, etc.; these spaces are not included in the built-up areas we took into consideration.

The fractal dimension is computed for a set of 26 **windows**. The size of the window (2750 x 2250 meters) is defined from the size and shape of the CBD of Brussels: it corresponds to the best fit by a rectangle to the Pentagon (CBD). The windows cover the total surface of the CBD (Figure 3); 10% overlapping of the windows is systematically applied horizontally and vertically in order to optimize the analysis of the spatial structure. A gliding window is then applied from left to right and from top to bottom. Due to the irregular administrative limits of the B.C.R., windows including at least 50% of B.C.R. are the only ones taken into account. Hence, 26 windows are kept in the analysis; each window receives an identification that corresponds to the x, y location on the grid proposed in **Fig. 2-Bru-1**. Given the characteristics of Brussels and the size of the windows, windows are – by definition – never homogeneous in terms of built-up patterns, or function. Ideally, smaller windows should lead to more homogeneous wards, but in such cases, they often reach the limit of the fractal software used.

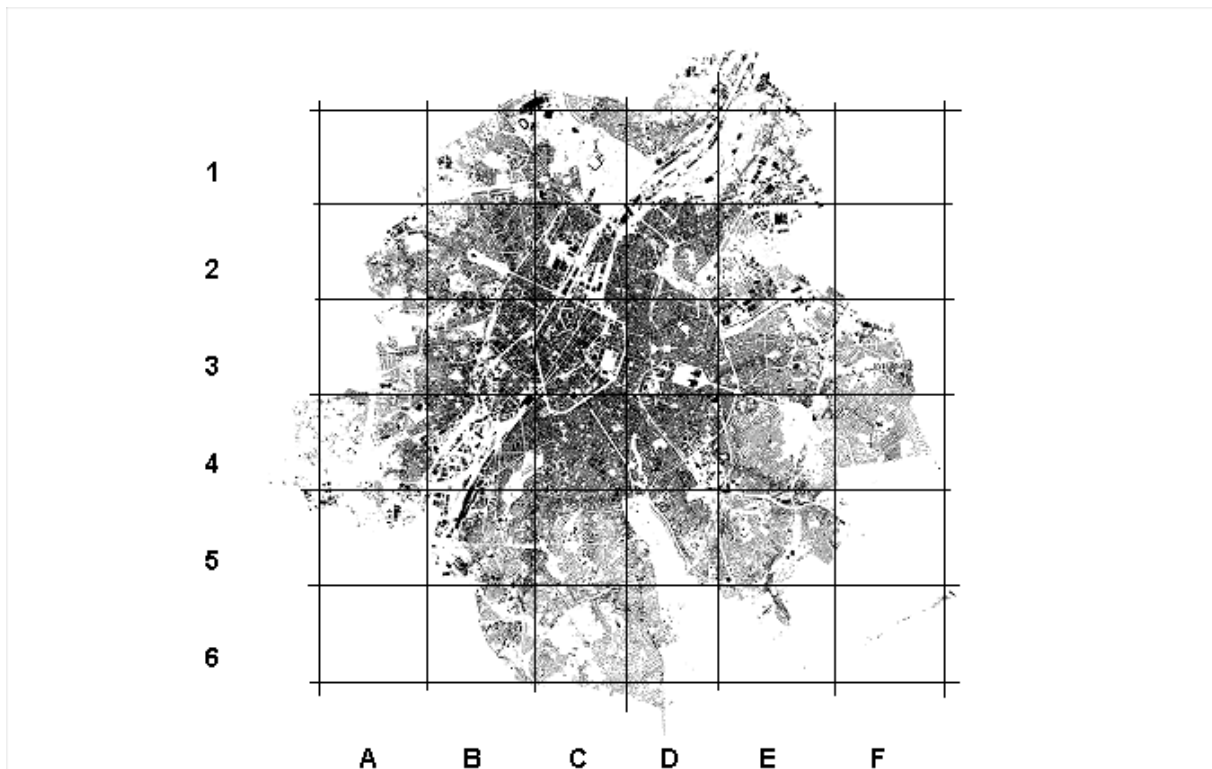


Fig. 2-Bru-1 *The built-up area of Brussels Capital Region*
Source: Brussels UrbIS

For each of the 26 windows, three fractal dimensions are computed: 2 pertain to the surfaces, one to the borders. Results are first analysed in a purely descriptive and exploratory way

(Section III) and then associated with other urbanistic and geographical variables (Section IV).

III. The fractal dimension of Brussels: a descriptive approach

The main objective of this paper is to conduct an exploratory data analysis on the values obtained for the 26 windows defined in Section II.c and to analyze the results in order to test whether fractal indices “measure” the urban landscape and give a good idea of the urbanistic history of the city. Two methods are applied: correlation (noted *Cor*) and dilation (noted *Dil*): they are applied to the surfaces of the built-up areas (noted *Surf*). For the dilation technique, several trials were made in terms of number of iterations; 30 dilation steps seems to be the best fit for most of the studied windows. Borders (lines) are also extracted by means of the dilation technique (5 iterations); the fractal dimension of the border is then computed using the correlation method (*Per*). *Cor* and *Dil* are expected to lead to different results as *Cor* is a second order and *Dil* a first order dimension (see Section II.b).

Whatever the technique used, the observed minimum value is 1.24 and the maximum 1.96. [Table 1](#) shows that – as expected – the values of D depend upon the method of estimation. Correlation analysis leads to larger D values than dilation; this can be explained by the above-mentioned artifact of estimation. In order to have global information about the different samples of windows, we also compute the arithmetic means of the D values. We are aware that from a statistical point-of-view, the fractal dimensions are not values that are really observed, since they are estimated by recurring to the values $N(\epsilon_j)$. Thus we might expect that a mean parameter should refer to these empirical values $N(\epsilon_j)$. However, since fractal dimensions are obtained by a non-linear estimation procedure over the used ranges of ϵ_i , we consider them as **independent synthetic measures** of the mass distribution in each window. This justifies the use of the arithmetic means of fractal dimensions as indicators for the order of magnitude of D .

It is obvious that the D values for a structure of linear topology (*Per*) are smaller than those referring to the occupied surface (*Surf*).

Table 1 - Descriptive statistics for D for different fractal methods

	$D_{Surf-Cor}$	$D_{Surf-Dil}$	$D_{Per-Cor}$
Mean	1.822	1.565	1.719
Minimum	1.338	1.261	1.543
Maximum	1.961	1.793	1.854

On average, whatever the method used, the highest values of D are observed in the city center, but the variation through the 26 windows differs according to the method used ([Table 2](#)). Measures referring to surfaces lead to D values positively and significantly associated in the space (+0.589), while the dimension of the border ($D_{Per-Cor}$) is negatively related to the dimension of the surfaces ($D_{Surf-Dil}$) or non significant ($D_{Surf-Cor}$). Let us add at this point that at this scale of analysis the border of the city has no or little sense. The perimeter is an artifact; it mainly measures the shape of the “non-built up areas” (lacunas). In our case, the denser the built-up areas, the smaller the green areas. All other Pearson’s correlation coefficients are not

significant. This suggests that each fractal estimation method measures a different component of the urban layout, and that the fractal behavior varies within the urban structure.

Table 2 - Pearson’s correlation coefficients between fractal dimensions D

	$D_{Surf-Cor}$	$D_{Surf-Dil}$
$D_{Surf-Dil}$	0.589	1.000
$D_{Per-Cor}$	n.s.	-0.458

As mentioned in Section II.b, let us now compare the fractal dimension D obtained by correlation on the built-up surfaces to that obtained on the perimeter. We know that (1) if D computed on surfaces (D_{Surf}) is equal to D computed on the perimeter (D_{Per}), we have a Sierpinsky Carpet; (2) if D_{Surf} tends to 2.0 and $D_{Per} < 2.0$, we have a teragon; and (3) if $D_{Surf} < 2.0$ and $D_{Per} \leq 2.0$ and different from D_{Surf} the structure is mixed. **Fig.2-Bru-2** gives the histograms of the D_{Surf} and D_{Per} values: they are all smaller than 2.0 and greater than 1.3. **Fig. 2-Bru-3** illustrates the relation (index) between D_{Surf} and D_{Per} : most values are smaller than 1.0, that is to say D_{Surf} is almost always smaller than D_{Per} . The index varies between 0.815 and 1.176: hence, the fractal structure is mixed.

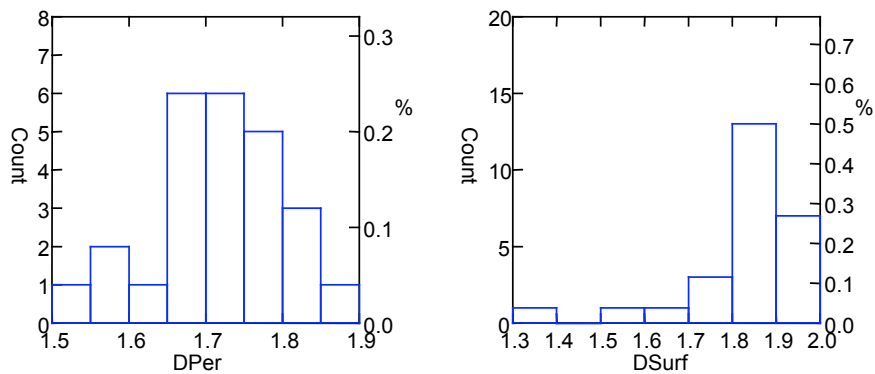


Fig.2-Bru-2 Histogram of $D_{Surf-cor}$ and $D_{Per-cor}$

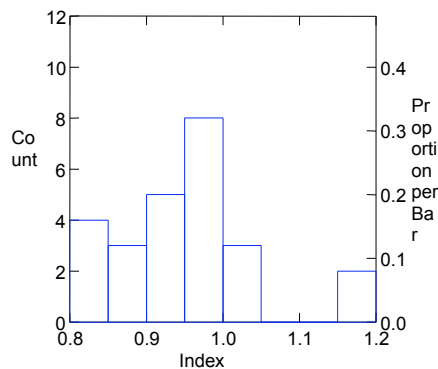


Fig. 2-Bru-3 Histogram of the index of $D_{Surf-cor}$ and $D_{Per-cor}$

Let us proceed to examine the extent to which measuring D conducts to a coherent urban structure, to a classification of the wards that reflects urbanization and land use planning that reflects former geographical analyses. A ward hierarchical clustering method was applied to the 26 windows, each window being characterized by three values of D ($D_{Surf-Cor}$, $D_{Surf-Dil}$ and $D_{Per-Cor}$). Distances were used for measuring resemblance. Again we can question

whether a classification of the dimensions is justifiable since the real observed values are the $N(\varepsilon)$. However, we adopt the same point-of-view as the preceding sub-section and consider that we dispose of a set of independent synthetic parameters D . Moreover, we show in **annexe C** that an estimation of the distances between fractal dimension values D is an appropriate measure for comparing scaling laws $N(\varepsilon)$. **Table 3** presents the results; it reveals a rather clear center-periphery structure: three sub-groups of windows are urban and two sub-groups of windows belong to the “suburban” part of the Brussels Capital-Region.

Table 3 - Results of the ward classification of 26 windows according to $D_{Surf-Cor}$, $D_{Surf-Dil}$ and $D_{Per-Cor}$

Group	Sub-group	Mean D			Content	Description
		$Surf-Dil$	$Surf-Cor$	$Per-Cor$		
Urban	U1	1.70	1.92	1.69	$B2, D2, C3, E3, C4, D4,$	City center and mixed wards of the 19 th century
	U2	1.75	1.89	1.65	$C2, B3, D3$	CBD with buildings and offices + some residential wards and old industrial wards
	U3	1.52	1.79	1.73	$C1, D1, B4, E4, B6, B1, E5$	Mixed residential parts of the 20 th century and industrial
Suburban	S1	1.46	1.82	1.78	$A2, E2, A3, F3, F4, C5, D5, C6$	Urban with gardens, 20 th century
	S2	1.42	1.44	1.68	$E1, A4$	Rural wards with large industrial surfaces and public equipment

The clustering of the windows reflects the history of the city (see e.g. De Keersmaecker and Carton, 1992). When Belgium became independent in 1830, Brussels was a market town, limited by ramparts. Its accession to the status of capital city of the country as well as the concomitant Industrial Revolution led to an important increase in population and hence a high pressure on its territory. The city became very dense in the center (behind the former ramparts) in an area called the Pentagon (here noted $C3$), and later extended to the periphery. At the end of the 19th century, large urban public works enabled further urban sprawl and new residential alternatives were offered to upper-class households in the East (Schaerbeek ($D2$), Woluwé-Saint-Lambert ($E3$) and Ixelles ($D4$)), in the South (Saint-Gilles ($C4$), Forest ($B5$)) and in the North (Koekelberg ($B2$)). All these wards belong to cluster U1 of the classification of the fractal dimensions. During the same period, the “Quartier Léopold” ($D3$) and the “Quartier Nord” ($C2$) developed respectively in the East and North of the city; those wards are centered on a main railway station and have been deeply restructured in the sixties by functionalistic urbanization. This is also the case of some parts of Molenbeek ($B3$), which originally developed around firms during the 19th century; hence, old council flats still characterize this part of the city. These wards belong to cluster U2.

20th century Man rejected the industrial city, and looked back to Nature. The expansion of the city in the Interbellum period (Cluster U3) was characterized by wards forming a first green belt. Several factors were behind this development: new tramway tracks, the Universal Exhibition in 1958 in the North of the city (Heysel – $B1$), and the construction of social housing by the public authorities (garden cities such as Floreal in Watermael Boitsfort). After

World War II, the city spread further and further away, “diluting” in the countryside; as in many other cities, this was due to the low price of land in the periphery, to the increasing use of the car and to the consumption of rural amenities. This characterizes the detached buildings of cluster S1 and the suburban wards of Uccle and Woluwé. Finally, in the western and northern parts of Brussels, there are still some rural spaces that are increasingly coveted and settled by industries, schools, hospitals, stores, or other private and public amenities. This characterizes cluster S2. Let us recall that “rehabilitation” is invisible at this level of analysis: it mainly affects the inside part of the buildings and not the relative organization of the buildings (no demolition).

From this descriptive exploratory data analysis, we can conclude (1) that each fractal dimension measures complementary aspects of the structure of the urban built-up area and (2) that the concomitant use of the three dimensions provides a rather accurate reflection of the intra-urban structure of Brussels. The urban structure seems to correspond to a multi-fractal logic OR to the overlay of different fractal patterns.

IV. A binary explanatory exploration of the data

Conclusions proposed in Section III confirm results obtained by models based on land-use characteristics and socio-economic and demographic statistics (see e.g. De Keersmaecker, 1990; Mort-Subite, 1990; Thomas and Zenou, 1999). Let us now test single bi-variate relationships between fractal dimensions and some variables often used in spatial analysis, that is to say geography and spatial economics.

IV.a Housing characteristics

Bid-rent theory postulates an implicit trade-off in housing decisions between housing space and type, and proximity to central urban functions. Housing type is hence a key variable for defining urban structure: cities are often articulated as spatial patterns with flats near the center, terraced houses occupying the inner suburbs and detached/semi-detached houses in the outer suburbs, each ring reflecting a stage in city growth. However, the date at which the land parcel was integrated into the urban development process also defines the structure of the neighborhood: a specific architectural project can lead to a specific spatial structure that should lead to specific and unique fractal dimensions. Hence, in this first hypothesis, we test the link between the fractal dimension D and some morphologic characteristics of the built-up type. A priori, this should be true for Brussels (see Section III), but this is difficult to measure as each window is – by definition – characterized by a mixed urbanization processes. In the case of a city such as Brussels, a homogeneous ward would be far too small to be analyzed by fractal methods. However, if the size of the analyzed areas is too small, the results are no longer reliable: the boundary effects become too important.

For each window, several variables were created in order to quantify the type of housing by available statistics. Most variables were obtained from the 1991 Population Census (I.N.S., 1991). Table 4 gives the relationship between the fractal dimensions and the housing characteristics. Given the density of the built-up areas, the dilation method leads to the most significant results close to those obtained by the correlation method. These are, however, very different from the dimension of the perimeter: it relates to the borders of the empty spaces (lacunas). Hence, if the mass is large, the theoretical number of possible arrangements is smaller than if there were only some small black dots to distribute. Hence, the variation of the potential fractal dimensions is smaller for the centers.

Table 4 - Pearson's correlation coefficients between fractal dimensions D and housing characteristics for several estimation methods

<i>Fractal dimension</i>	$D_{Surf-Cor}$	$D_{Surf-Dil}$	$D_{Per-Cor}$
Housing characteristic			
% detached and semi-detached housing	-0.841	-0.713	n.s.
% dense terraced housing	-0.345	n.s.	0.473
% flats /total housing	0.84	0.72	n.s.
% flats in buildings of more than 10 housings	n.s.	0.504	n.s.
Concentration of buildings of more than 10 housings	-0.37	n.s.	0.37
Number of flats/number of houses	0.86	0.57	-0.44
Number of flats in buildings of more than 10 housings/number of detached housings	0.73	0.56	n.s.

(n.s. = not significant at $\alpha = 0.05$)

Our first conclusion states that each estimation technique (Dil , Cor) conducts to a value of D that is significantly related to the characteristics of the housing. Further linear regression analyses (not reported here) showed that more than 85% of the variation of D can be explained by the type of housing (by the % of detached housing and by the index of the number of flats divided by the number of houses). Moreover, land uses that are larger in scope (i.e. residence) have a greater degree of irregularity simply because they are larger in scale and there is hence less effort put into the geometric control on land under development. This second statement confirms Batty and Longley's results (1994, page 226). Last but not least, given the different nature of the fractal estimation methods applied here, differences in the relationships are observed especially when comparing surfaces to borders; note the opposition in sign between the correlation coefficients computed for surfaces and that for perimeters.

Let us also mention that the history of the development of the city also influences the fractal dimension. [Table 5](#) gives the relationship between the age of the housing and D . Pearson's correlation coefficients are never significant for the fractal correlation method computed for surfaces. $D_{Surf-Dil}$ and $D_{Per-Cor}$ give more significant results, but the sign of the Pearson's correlation coefficient is often opposite: $D_{Surf-Dil}$ refers to the built-up areas (surfaces), $D_{Per-Cor}$ refers to the shape of the lacunas, the spaces left empty between the built-up areas. The older the housing, the more dense the urbanization and the higher $D_{Surf-Dil}$ (the smaller $D_{Per-Cor}$). To the contrary, the more recent the housing, the larger it is (further away from the city center) and hence the higher $D_{Per-Cor}$ and the smaller $D_{Surf-Dil}$.

Table 5 - Pearson's correlation coefficient between fractal dimensions and the age of the housing for several estimation techniques

<i>Fractal dimension</i>	$D_{Surf-Cor}$	$D_{Surf-Dil}$	$D_{Per-Cor}$
Age characteristics			
% housing built before 1919	0.609	n.s.	-0.655
% housing built before 1945	0.680	n.s.	-0.606
% housing built after 1960	-0.492	n.s.	0.586
% housing built after 1971	-0.481	n.s.	0.507
Mean age of the housing	-0.647	n.s.	0.640
Housing after 1961/housing before 1945	0.500	n.s.	-0.585

Note: n.s.: not significant at $\alpha = 0.05$

Let us recall that Pearson's correlation coefficient between the age of the buildings and $D_{Surf-Cor}$ (Table 5) is never significant at the 5% level. Their values, however, vary according to the age of the buildings (not reported here). In fact, dilation of the image ($D_{Surf-Dil}$) implies dilating the built-up surfaces and hence ignoring the irregularities, the details, the "noises" observed in the built-up areas. Hence, this technique (*Dil*) tends to a generalization, to a better "modeling" of the built-up areas. To the contrary, $D_{Surf-Cor}$ is computed on built-up surfaces that are not dilated, and hence takes into account more irregularities. They are hence non-significantly related to the average measures of age computed on wards.

IV.b Distance to the CBD

Distance is a key factor for interpreting the internal structure of a city (Anas et al., 1998): on average, the further from the CBD, the most recent the residence, the smaller the rent, the larger the houses and the higher the transportation costs. This is due to the fact that it is possible nowadays to reside in a nice natural environment far from urban stress, in large houses with gardens, while the sources of income and employment remain in the city center (see e.g. Le Jeannic, 1997; Goffette-Nagot, 2000; Cavailhès *et al.*, 2001). Hence, we expect a significant difference between the fractal dimension in the city center compared to that of the outskirts.

Table 6 gives the Pearson's correlation coefficients between several computations of the fractal dimension D and the crow-fly distance to the city center. Two ways of defining the city center are used: one corresponds to the historical and administrative core of the city (Grand-Place), the other corresponds to the location of offices and many national and international administrations (Quartier Léopold). As expected, dimensions computed for surfaces ($D_{Surf-Cor}$, $D_{Surf-Dil}$) lead to significant negative coefficients: the center is denser and more homogeneous in terms of built-up areas. The dilation method, however, leads to higher values of the Pearson correlation coefficients. As expected, dimensions computed for the lacunas give opposite results: the greater the distance to the center, the higher the value of $D_{Per-Cor}$; borders are better delineated when "green areas" (gardens, etc.) are larger. This is the case for the windows located in the periphery of the studied area.

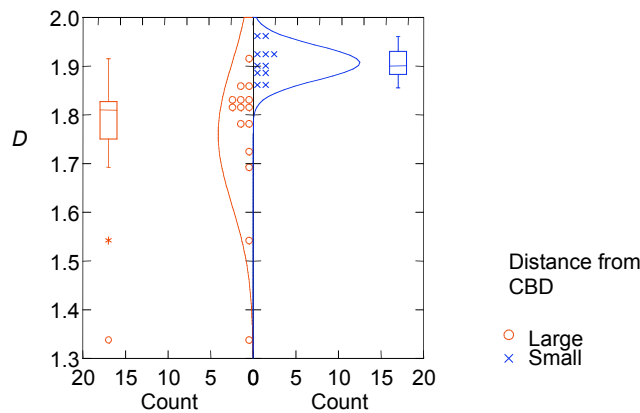
In the preceding subsection (IV.-a) it was shown that fractal dimensions are related to the stage of development of the city (history); this subsection (IV.-b) confirms this fact as it shows that fractal dimensions are significantly related to the distance to the CBD.

Table 6 - Pearson's correlation coefficients between D and distance to CBD, for two ways of defining the center and several estimation techniques

<i>dimension</i>	Fr	$D_{Surf-Cor}$	$D_{Surf-Dil}$	$D_{Per-Cor}$
Distance to Grand-Place		-0.623	-0.935	0.525
Quartier Léopold		-0.716	-0.844	n.s.

Figure 6 translates graphically the results of the comparisons of the $D_{Surf-Cor}$ values for large and small distances from the CBD (larger/lower than 1.800 meters). The absolute values of D are greater near the center, and their variation is smaller than further away from the city center: centers are more homogeneous and more densely built. More important fluctuations at greater distances from the CBD (LHS of **Fig. 2-Bru-4**) mean that the spatial organization is weaker at the outskirts and hence the complexity of the built-up areas is greater: on average, housing in the periphery is not planned and its structure is spatially more heterogeneous. D is a measure of diversity.

Fig. 2-Bru-4 Distribution of the $D_{Surf-Cor}$ values for windows close to the city center (RHS) and windows further away (LHS)



IV.c Rent

From spatial economics, we know that a household selects a residential location within a metropolitan area that is made up of several residential sites. The household's fixed income per unit of time is assigned to hiring a residential plot, commuting to the CBD and enjoying urban and rural amenities (see e.g. Cavailhès et al., 2002 and 2003). In this case, we wanted to test whether this economical mechanism could be translated into the built-up characteristics and hence by fractal dimensions. In Brussels, we know that high rents are associated to periurban locations as far as residence is concerned, and to central locations for offices (see e.g. Goffette-Nagot, Thomas and Zenou, 2000).

Three rent indicators are considered here. The first is the average monthly rent for housing obtained by an annual survey (De Keersmaecker, 1994). The second is the population density, which should give an indication of the intensity of the pressure on real estate. Thirdly, human density is considered; it refers to population plus employment density.

Table 7 - Pearson's correlation coefficients between D and rent for several fractal estimation techniques

<i>Fractal dimension</i> Rent	$D_{Surf-Cor}$	$D_{Surf-Dil}$	$D_{Per-Cor}$
Average rent for housing	n.s.	-0.680	0.533
Population density	0.723	0.834	-0.405
Population + employment density	0.648	0.869	-0.533

Table 7 reveals quite interesting results: first, it shows that fractal dimensions related to built-up surfaces ($D_{Surf-Cor}$ and $D_{Surf-Dil}$) increase when density increases, confirming that large population densities are associated to homogeneous built-up areas (D large) and inversely. This also means that low densities have a different spatial structure of the built-up areas: in terms of planning and land uses, small densities mean less buildings and hence a greater variety in the size of the empty spaces. This confirms the classical North American model: a center with dense, rather small, uncomfortable and low-quality housing and a periphery with larger housing, less dense and rather comfortable.

Another way of interpreting these results is to refer to former geographical analyses: results are simply to be related to the structure of the housing market. Indeed, former papers (see e.g. De Lannoy and Kesteloot, 1985; Vanderhaegen et al., 1996; Kesteloot, 1997) have shown that in Brussels the housing market is structured into three concentric circles of decreasing rent. A first set of central wards dating mainly from the 19th century where housing mainly corresponds to old houses divided into flats occupied by tenants; some were renovated but most were not, explaining the low central rents for housings. The second concentric zone, is – on average – characterized by more recent housing of better quality. In this zone, we also find some wards characterized by social housing; in some of them, the housing has been sold to the private sector. In the third zone, most of the housing is occupied by the owner; housing consists mainly of detached or semi-detached houses or high standard flats.

Whatever the theoretical background for interpreting the results, fractal dimensions are significantly related to the rental housing market.

IV.d Household income

For Brussels, we know that on the average, high incomes are located in the periphery and poor people characterize the central wards (see e.g. De Keersmaecker and Carton, 1995; Vanderhaegen et al., 1996; Thomas and Zenou, 1999 or Goffette-Nagot, Thomas and Zenou, 2000). Income is a way to appropriate the socio-economic characteristics of the inhabitants. Median, average and interquartile income are available in Belgium at the scale of the urban ward, for each urban ward containing at least 30 households (I.N.S., 1994). A synthetic value

is then computed for each window. Pearson's correlation coefficients between D values and income are presented in [Table 8](#).

Table 8 - Pearson's correlation coefficients between D and household income, for several fractal estimations

<i>dimension</i>	$D_{Surf-Cor}$	$D_{Surf-Dil}$	$D_{Per-Cor}$
Income			
Average	n.s.	-0.743	0.496
Median	n.s.	-0.755	0.569
Interquartile difference	n.s.	-0.758	0.457

n.s. = not significant at 5% level

D obtained by correlation on the built-up surfaces ($D_{Surf-Cor}$) is never significantly related to income. Dilation applied to surfaces ($D_{Surf-Dil}$) produces D values negatively and significantly related to income, whatever the way of measuring the income: high values of $D_{Surf-Dil}$ are associated to small incomes reflecting the particular structure of Brussels, where rich households are located at the outskirts and poor people occupy the densely built-up areas of the city center (see structure of the housing market in IV.-c). As expected, the correlation method applied to the perimeter leads to opposite results: the higher the income, the higher $D_{Per-Cor}$, the greater the sizes and regularity of the gardens and empty spaces. Once again, [Table 8](#) cannot be interpreted on its own: a tight link is to be made (1) with the history of the city, its development and the resulting structure of the built-up areas, and (2) with the spatial structure of the housing market and rents.

IV.e A multivariate explanatory attempt

Most variables used in the preceding subsections are now introduced in a multivariate regression approach. The objective here is not to produce a predictive urban model; the choice of the dependent and independent variables therefore need much more discussion. In this sense, we can even wonder if the socioeconomic variables “explain” fractal dimension or inversely? Our aim here is simply to see how far the variation of the fractal dimensions are associated to the co-variations of rent, distance, housing structure, etc. commonly used in urban models. Our objective is purely exploratory. Each dimension ($D_{Surf-Cor}$, $D_{Surf-Dil}$ and $D_{Per-Cor}$) is hence analyzed separately and considered as a dependent variable (Y_i). “Explanatory variables” (X_i) are highly inter-correlated; hence, a factor analysis followed by a Varimax rotation is applied to these variables and the rotated factor scores are introduced as explanatory variables in linear O.L.S. stepwise regressions. Results are summarized in [Table 9](#).

Table 9-a - **Multivariate results: rotated loading matrix**

	<i>Factor 1</i>	<i>Factor 1</i>	<i>Factor 3</i>
% detached housing	0.007	0.682	0.455
% terraced housing	0.148	0.725	0.023
% buildings > 10 housings	0.575	-0.773	-0.044
% housing built before 1919	-0.823	-0.208	-0.322
% housing built before 1945	-0.884	-0.258	-0.270
% housing built after 1960	0.976	0.011	0.117
% housing built after 1971	0.872	0.056	0.020
Mean age of housing	0.931	0.195	0.254
Index of oldness	0.959	0.017	0.113
Pop. + employment density	-0.471	-0.758	-0.293
Average income	0.280	0.236	0.880
Central rent	0.126	0.129	0.935
Total rent	0.169	0.200	0.928
Distance to center	0.401	0.762	0.434

Table 9-b - **Multivariate results: Pearson's correlation coefficients**
(n.s. = not significant at 0.10)

	$D_{Surf-Cor}$	$D_{Surf-Dil}$	$D_{Per-Cor}$
Factor 1	n.s.	n.s.	0.573
Factor 2	-0.778	-0.630	n.s.
Factor 3	n.s.	-0.543	n.s.

Table 9-c - **Multivariate results: regression equations (coefficients significant at 0.05)**

	$D_{Surf-Cor}$		$D_{Surf-Dil}$		$D_{Per-Cor}$	
	Coeff. (in %)	R_{-}	Coeff. (in %)	R_{-}	Coeff. (in %)	R_{-}
Intercept	1.822		1.565		1.719	
Factor 1			-0.058 16.8%		0.048 35.7%	
Factor 2	-0.094 49.4%		-0.099 48.6%		0.023 31.0%	
Factor 3			-0.067 22.6%			
R_{-}		49.4%		88.0%		66.7%

Three factors summarize 86% of the initial information (eigenvalue > 1.0). *Factor 1* mainly translates the characteristics of the age of the housing. *Factor 2* groups the variables measuring density and *Factor 3* is mainly an economic factor grouping variables related to income and rent. Pearson's correlation coefficients already reveal the differences between the variations: D related to the border ($D_{Per-Cor}$) is mainly related to the age of the housing (*Factor 1*). In fact, the more recent the housing, the larger the gardens and green spaces and the more detailed the planning, and hence the "better" the border is delineated. Fractal dimensions related to surfaces ($D_{surf-cor}$, $D_{surf-dil}$) are best related to *Factor 2* and *Factor 3*, to respective density measures (*Factor 2*) and to the economic characteristics of the wards (*Factor 3*).

The scores of the three components now become the independent variables of the fractal dimensions (O.L.S. regression) (Table 9-c). The dimension obtained by dilation ($D_{surf-dil}$) is best "explained": all three factors enter the equation significantly, leading to a coefficient of determination of 88.0%. Each factor score is negatively related to the fractal dimension. The higher the factor scores, the smaller the dimension. As mentioned previously, $D_{surf-dil}$ is in fact computed on an image that is smoothed by dilation; many spatial details are hence lost, leading to less noise and to statistically better results. To the contrary, dimension computed on surfaces by the correlation method ($D_{surf-cor}$) is computed on an image that is not smoothed and the explanation of its variation is quite different and much lower: R_{-} is equal to 49% and D is negatively associated with the variation of only one factor (*Factor 2*) which is a measure of density. The higher the factor scores, the lower the value of $D_{surf-cor}$. Finally, the fractal dimension of the perimeter is positively associated with the age of the housing (*Factor 1*) and with density measures (*Factor 2*): the lower the density and the more recent the houses, the better the perimeter is delineated.

This confirms preceding comments: distance to the CBD translates the center-periphery structure of the city not only in terms of the rent of housing and household income, but also in terms of the history of the urbanization procedure. Hence, the fractal dimension is an indicator

of the morphology of the urban ward, but each method for estimating this dimension brings complementary information.

V. Conclusions and perspectives

The objective of this paper was to compare fractal-based parameters computed by different fractal analysis methods applied on urban built-up areas in Brussels, and to explain the observed spatial variations by means of variables commonly used in geography, urban economics and land use planning. Bearing in mind the limited and exploratory nature of the analysis undertaken here, it is possible to state the following conclusions:

- (1) With one intra-urban example (Brussels), this paper confirms that with **fractal analysis**, one can quite reasonably describe the spatial dilution of the built-up areas within a metropolitan area. Such analysis describes the structures of the built-up surfaces as well as the limits of the green spaces or lacunas; the fractal descriptors are a priori quite complex but take into account the hierarchical nature of the urbanization process. It is more than a measure of density: density does not take the spatial structure into account.
- (2) The paper confirms the **sensitivity** of the results to the fractal method used. More particularly, it shows that the structure of the intra-urban built-up areas seems to correspond to a **multi-fractal** logic OR to the overlay of different fractal patterns. The different exploratory data analyses show that the fractal dimensions vary differently and fractal characterization may hence lead to false results when applying only one method. Methods related to the surfaces provide good indicators of the spatial internal structure of the built-up areas; those related to the borderlines give interesting results on the structure and shape of the green areas (e.g. gardens). Methods related to surfaces provide results that co-vary in space but are not equal. Fractal dimensions describe the heterogeneity of the spatial distribution of lines (limits), surfaces or volumes. It gives information on the division of the zone; it measures the structure of the built-up area according to its complexity and its dimensional behavior (tending respectively to a line or to a surface). It gives information on its shape and on its hierarchical structure (nested scales). Fractal dimension is hence a better descriptor than population density: it takes into account the underlying structure of the built-up areas.
- (3) Interesting **statistical associations** can be found with the structure of the housing market, the rent, the distance to the city center, the income of the households as well as some planning rules. Given its nature in time and space, housing market and distance to the CBD have a strong center-periphery organization that is to be translated by the computed fractal dimensions. We know that for Brussels, however, the structure is the inverse of Paris: Brussels follows the North American type of urban model. The associations put forward in this paper strongly confirm the interpretation of the urban structure; this is done within a geographical, economical and historical background. Hence, coupled with an adequate model, fractal simulation will definitely improve the functioning of the city. It “describes” the structure, but is also a powerful instrument for analyzing and planning.

The analyses performed in this paper are suggestive and exploratory rather than definitive. They reveal some basic problems of observation and measurement that are generic to most empirical science. We are aware that our results are only valid in the scope of this application for Brussels. Results are limited by the availability of the data and by the different choices made for defining the built-up areas and the windows. However, the results reported here are relevant and confirm other empirical and modeling approaches used for Brussels. Moreover,

they are included in a larger project that will lead to the comparison of several European cities; at this stage of the comparison, results seem to converge.

Despite the limits of the case study as well as the limits of the method itself, fractal analysis seems to be a promising tool for describing the morphology of the city and for understanding its genesis and planning. Fractals are far from being an explanatory tool (and will never be!), but they do seem to be a good tool for reproducing urban inner morphology, for simulating. The model seems to be – on average – robust : it replicates the urban spatial regularities and patterns, and could hence be fruitfully integrated at a later stage in intra-urban simulation processes (saving time and money!).

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